

Generalized Entropy and Transport Coefficients of Hadronic Matter

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Abstract. We use the generalized entropy four-current of the Müller-Israel-Stewart (MIS) theories of relativistic dissipative fluids to obtain information about fluctuations around equilibrium states. This allows one to compute the non-classical coefficients of the entropy 4-flux in terms of the equilibrium distribution functions. The Green-Kubo formulae are used to compute the standard transport coefficients from the fluctuations of entropy due to dissipative fluxes.

1 Introduction

Transport coefficients, such as viscosities, diffusivities, and conductivities characterizes the dynamics of fluctuations in a system. Transport phenomenon in relativistic fluids are of great interest in connection with the study of astrophysical conditions such as those in neutron stars, the study of the cosmological conditions such as those in the early universe, as well as the study of the relativistic nuclear collisions such as those at the Relativistic Heavy Ion Collider (RHIC) and at the Large Hadron Collider (LHC).

Knowledge of transport coefficients and associated lengths and/or time scales is important in comparing observables with theoretical predictions. In the study of relativistic nuclear collisions knowledge of transport coefficients will greatly advance our current efforts and interests in the use of relativistic dissipative fluid dynamics in describing the observables [1,2,3,4]. Relativistic heavy ion collisions offer the opportunity to study interactions between hadrons over a wide range of net baryon density (or baryon chemical potential) and energy density (or temperature). In order to study the transport properties of hadronic matter produced in such collisions one should extract the transport coefficients and associated length/time scales for a given model of interacting hadrons. Then using relativistic dissipative fluid dynamics one can study the sensitivity of the space-time evolution of the system and the calculated distributions of the hadrons to dissipative, non-equilibrium processes. In the end one needs to compare the predicted distributions with those observed in experiments.

2 Second-order entropy 4-current in MIS theories

In the MIS theories of relativistic fluid dynamics the entropy 4-current is a function of both the classical variables and dissipative fluxes, i.e., $S^\mu = s^\mu(\varepsilon, n, u^\mu, \Pi, q^\mu, \pi^{\mu\nu})$ where ε is the energy density, n is the net charge, u^μ is the 4-velocity, Π is the bulk viscous pressure, q^μ is the

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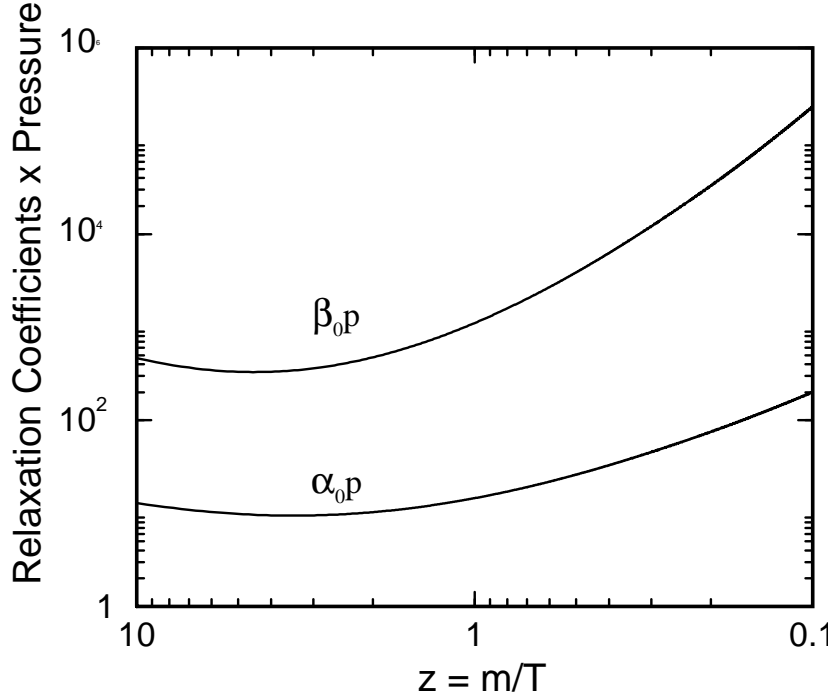


Fig. 1. The temperature dependence of the relaxation/coupling coefficients, α_0 and β_0 for a pion gas.

heat flux and $\pi^{\mu\nu}$ is the shear viscous pressure tensor. The second order or generalized entropy 4-current in the MIS theories of relativistic dissipative fluids [5] may be written as [6,7]

$$S^\mu = s_{\text{eq}} u^\mu + \beta q^\mu - \frac{1}{2} \beta u^\mu (\beta_0 \Pi^2 - \beta_1 q^\nu q_\nu + \beta_2 \pi^{\nu\lambda} \pi_{\nu\lambda}) - \beta (\alpha_0 q^\mu \Pi - \alpha_1 q_\nu \pi^{\mu\nu}) \quad (1)$$

where $s_{\text{eq}}(\varepsilon, n)$ is the equilibrium entropy density, u^μ is the hydrodynamical 4-velocity of the net charge and is to be normalized such that $u^\mu u_\mu = 1$, $\beta \equiv 1/T$ is the inverse temperature. The non-classical coefficients $\alpha_i(\varepsilon, n)$ and $\beta_i(\varepsilon, n)$ in Eq. (1) are expressed in terms of thermodynamic integrals or can be obtained as differentiations of the equation of state $p \equiv p(\alpha, \beta)$ with respect to α and β (where $\alpha \equiv \beta\mu$ with μ the chemical potential). The transition to $p \equiv p(\varepsilon, n)$ can be done through the relations

$$n = \beta \frac{\partial p}{\partial \alpha}, \quad \varepsilon = - \left(p + \beta \frac{\partial p}{\partial \beta} \right). \quad (2)$$

These non-classical second order coefficients are presented in detail in Ref. [7]. From the entropy 4-current, Eq. (1), the entropy density and flux are, respectively, given by

$$s = u_\mu S^\mu = s_{\text{eq}}(\varepsilon, n) - \frac{1}{2} \beta (\beta_0 \Pi^2 - \beta_1 q_\nu q_\nu + \beta_2 \pi^{\nu\lambda} \pi_{\nu\lambda}), \quad (3)$$

$$\Phi^\mu = \Delta^{\mu\nu} S_\nu = \beta q^\mu - \beta (\alpha_0 q^\mu \Pi - \alpha_1 q_\nu \pi^{\mu\nu}), \quad (4)$$

where $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the projection tensor with $g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ the Minkowski metric tensor. The entropy density $s \equiv s(\varepsilon, n, u^\mu, q^\mu, \Pi, \pi^{\mu\nu})$ is independent of the α_i while the entropy flux is independent of β_i . The thermodynamic coefficients $\beta_i(\varepsilon, n) \geq 0$ in Eq. (3) model deviations of the physical entropy density s due to dissipative contributions to S^μ . The $\alpha_i(\varepsilon, n)$ in Eq. (4) model contributions due to viscosity/heat couplings, which do not influence the physical entropy.

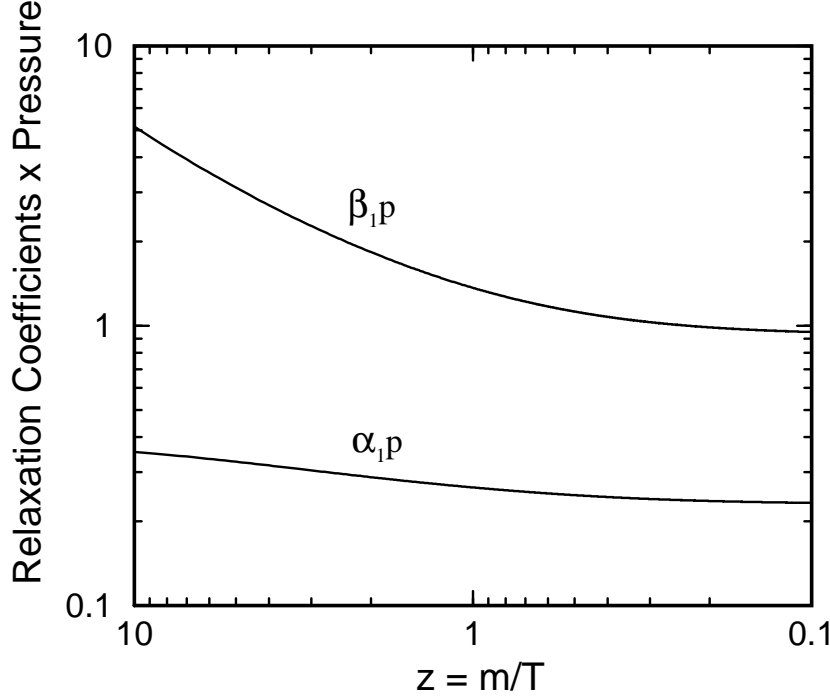


Fig. 2. The temperature dependence of the relaxation/coupling coefficients, α_1 and β_1 , for a pion gas.

The non-classical coefficients α_i and β_i are shown in Figs. 1, 2 and 3. Shown in these figures are these coefficients, multiplied by pressure so that they are dimensionless, as functions of the ratio of pion mass to temperature $z = m/T$. These non-classical coefficients are not parameters but they are constrained by the equation of state. Thus to compute the entropy one needs just the knowledge of either the standard transport coefficients (in the case of a simple one component fluid these are the bulk and shear viscosities and the heat conductivity), or the corresponding relaxation times to determine the dissipative fluxes.

3 Second moments of equilibrium fluctuations

We shall now use the generalized entropy discussed in the previous section to obtain the transport coefficients (cf. [8]). In the local rest frame of the 3+1 formulation [6] the entropy density may be written as

$$s(\varepsilon, n, \mathbf{q}, \Pi, \boldsymbol{\pi}) = s_{\text{eq}}(\varepsilon, n) - \frac{1}{2}\beta (\beta_0 \Pi^2 + \beta_1 \mathbf{q} \cdot \mathbf{q} + \beta_2 \boldsymbol{\pi} : \boldsymbol{\pi}) \quad (5)$$

The generalized Gibbs equation then takes the form

$$ds = \frac{\partial s}{\partial \varepsilon} d\varepsilon + \frac{\partial s}{\partial n} dn + \frac{\partial s}{\partial \mathbf{q}} \cdot d\mathbf{q} + \frac{\partial s}{\partial \Pi} d\Pi + \frac{\partial s}{\partial \boldsymbol{\pi}} : d\boldsymbol{\pi} \quad (6)$$

The probability W of fluctuations of thermodynamic variables with respect to their equilibrium reference values is related to the second differential of the entropy by the Einstein formula [10] for probability of fluctuations

$$W \approx \exp \left[\frac{1}{2} \delta^2 S \right] , \quad (7)$$

where S is the total entropy of the system in volume V . For small fluctuations around equilibrium states the change in entropy $\Delta S = S - S_{\text{eq}}$ can be expanded as

$$\Delta S \approx (\delta S)_{\text{eq}} + \frac{1}{2}(\delta^2 S)_{\text{eq}}. \quad (8)$$

Since the entropy is maximum in equilibrium, $(\delta S)_{\text{eq}} = 0$ and $(\delta^2 S)_{\text{eq}} \leq 0$. The second differential of the generalized entropy may be derived from the Gibbs equation, Eq. (6), and when the resulting expression is introduced into Eq. (7) one obtains

$$W(\delta\varepsilon, \delta n, \delta\Pi, \delta\mathbf{q}, \delta\boldsymbol{\pi}) \approx \exp\left\{\frac{1}{2}V\left[\frac{\partial^2 s_{\text{eq}}}{\partial\varepsilon^2}(\delta\varepsilon)^2 + 2\frac{\partial^2 s_{\text{eq}}}{\partial\varepsilon\partial n}\delta\varepsilon\delta n + \frac{\partial^2 s_{\text{eq}}}{\partial n^2}(\delta n)^2 - \beta(\beta_0\delta\Pi\delta\Pi + \beta_1\delta\mathbf{q} \cdot \delta\mathbf{q} + \beta_2\delta\boldsymbol{\pi} : \delta\boldsymbol{\pi})\right]\right\} \quad (9)$$

The second moments of a multi-variant distribution function

$$W \approx \exp\left[-\frac{1}{2}M_{ij}\delta x_i\delta x_j\right], \quad (10)$$

where M_{ij} is the matrix corresponding to the second derivatives of the entropy with respect to the classical and flux variables, are given by

$$\langle\delta x_i\delta x_j\rangle = (M^{-1})_{ij}, \quad (11)$$

where the brackets $\langle\dots\rangle$ denote the average over the probability distribution. In equilibrium the fluctuations of the classical variables ε and n are uncoupled with the fluctuations of the fluxes. The second moments of the fluctuations of Π , \mathbf{q} and $\boldsymbol{\pi}$ are given by

$$\begin{aligned} \langle\delta\Pi\delta\Pi\rangle &= \frac{T}{\beta_0 V}, \\ \langle\delta q_i\delta q_j\rangle &= \frac{T}{\beta_1 V}\delta_{ij}, \\ \langle\delta\pi_{ij}\delta\pi_{kl}\rangle &= \frac{T}{\beta_2 V}\Delta_{ijkl}, \end{aligned} \quad (12)$$

with δ_{ij} the Kronecker symbol and $\Delta_{ijkl} = (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl})$.

Recall that the classical coefficients of bulk viscosity ζ , thermal conductivity κ and shear viscosity η are related to the second order non-classical coefficients β_i by [7] $\beta_0 = \tau_\Pi/\zeta$, $\beta_1 = \tau_q/\kappa T$ and $\beta_2 = \tau_\pi/2\eta$ where the τ_A are the relaxation times for the respective dissipative fluxes. Thus the expressions Eqs. (12) relate the dissipative coefficients ζ , κ and η to the fluctuations of the fluxes with respect to equilibrium. Hence these coefficients determine the strength of the fluctuations or vice versa the fluctuations determine the dissipative coefficients. The expressions, Eqs. (12) for the second moments of the fluctuations allow us to compute the dissipative coefficients of the generalized entropy expression from the equilibrium distribution function and microscopic expressions of the dissipative fluxes.

The Green-Kubo relations for bulk viscosity, thermal conductivity and shear viscosity can be written as

$$\begin{aligned} \zeta &= \frac{V}{T} \int_0^\infty \langle\delta\Pi(0)\delta\Pi(t)\rangle dt, \\ \kappa &= \frac{V}{T^2} \int_0^\infty \langle\delta q_i(0)\delta q_j(t)\rangle dt, \\ \eta &= \frac{V}{T} \int_0^\infty \langle\delta\pi_{ij}(0)\delta\pi_{ij}(t)\rangle dt \quad (i \neq j), \end{aligned} \quad (13)$$

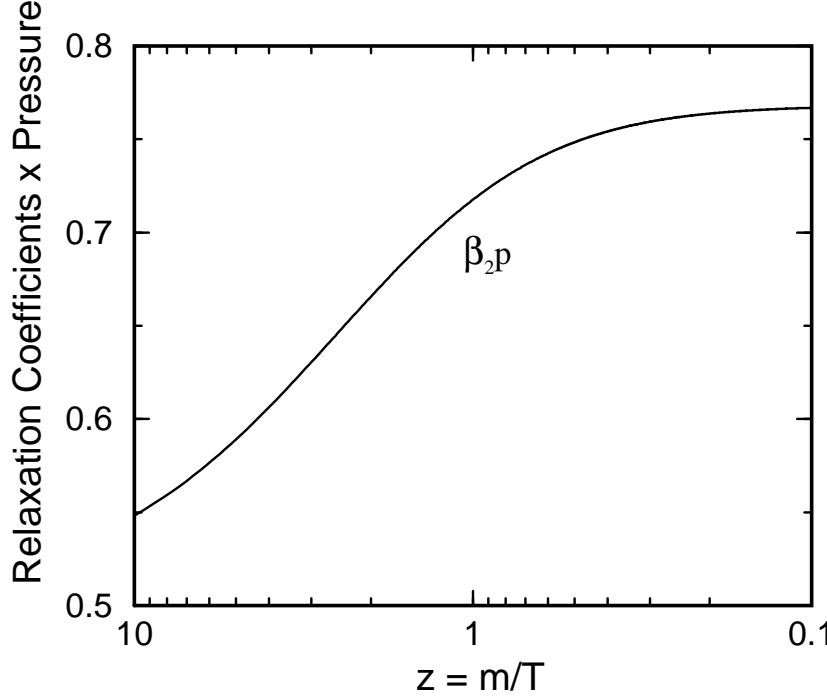


Fig. 3. The temperature dependence of the shear viscous relaxation coefficient, β_2 , for a pion gas.

respectively. In Eqs. (13) the brackets $\langle \dots \rangle$ stand for equilibrium average, and no summation is implied over the repeated indices. The time correlation functions can be calculated from Eq. (12) together with Maxwell-Cattaneo relations [9] as presented in Ref. [6] to give

$$\begin{aligned} \langle \delta \Pi(0) \delta \Pi(t) \rangle &= \zeta T (\tau_\Pi V)^{-1} \exp(-t/\tau_\Pi) , \\ \langle \delta q_i(0) \delta q_j(t) \rangle &= \lambda T^2 (\tau_q V)^{-1} \delta_{ij} \exp(-t/\tau_q) , \\ \langle \delta \pi_{ij}(0) \delta \pi_{kl}(t) \rangle &= \eta T (\tau_\pi V)^{-1} \Delta_{ijkl} \exp(-t/\tau_\pi) , \end{aligned} \quad (14)$$

If the evolution of the fluctuations of the fluxes is described by Maxwell-Cattaneo relaxation equations (cf [7]), then after integration, Eqs. (13) reduce to

$$\zeta = \tau_\Pi \frac{V}{T} \langle \delta \Pi(0) \delta \Pi(0) \rangle \quad (15)$$

$$\kappa = \tau_q \frac{V}{T^2} \langle \delta q_i(0) \delta q_i(0) \rangle , \quad (16)$$

$$\eta = \tau_\pi \frac{V}{T} \langle \delta \pi_{ij}(0) \delta \pi_{ij}(0) \rangle . \quad (17)$$

4 Hadronic Matter

In spite of their importance, transport coefficients of hot, dense hadronic gases are still difficult to calculate from first principles. Progress in the study of hadronic matter transport coefficients is very slow, and only a calculation of transport coefficients in the variational method [11,12] and relaxation time approximation [13] has been done. Though these models can describe some aspects of the properties of the hadronic matter, whether they are realistic enough or not is unclear. Thus, we need to investigate the transport properties of a hadron gas by using a microscopic model that includes realistic interactions among hadrons. This is of great interest in relativistic nuclear collision [14,15]. In this work, we adopt a relativistic microscopic model,

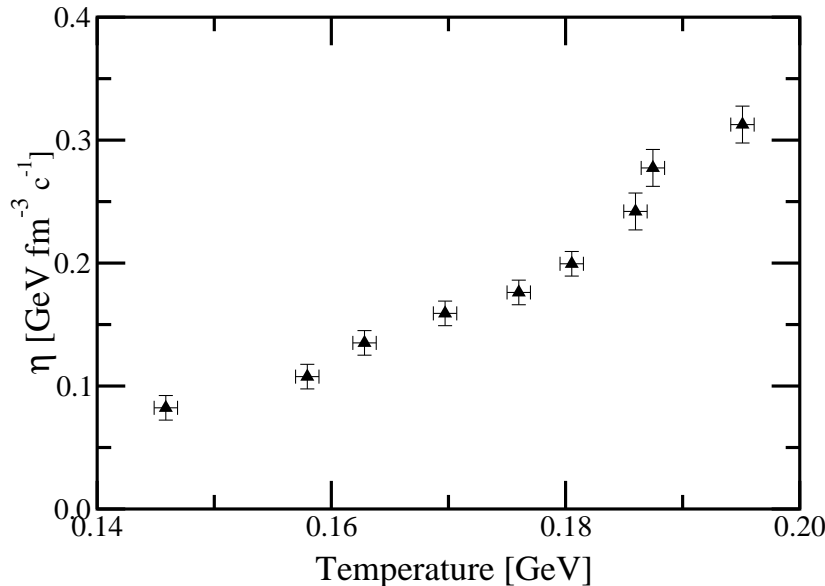


Fig. 4. Shear viscosity of meson gas as a function of temperature (cf. [14]).

UrQMD [16] and perform molecular-dynamical simulations for a hadronic gas of mesons in a box of volume V .

We focus on the hadronic scale temperature ($100 \text{ MeV} < T < 200 \text{ MeV}$) and zero baryon number density which are expected to be realized in the central high energy nuclear collisions. Transport coefficients of hadronic matter in this region should play important roles in phenomenological models. Sets of statistical ensembles are prepared for the system at different energy densities. Using these ensembles, the shear viscosity coefficient of a hadronic gas of mesons is studied as a function of temperature.

In computing the non-classical coefficients appearing in the generalized entropy, Eq. (1) we consider a system of single component gas of pions. In Figs. 1, 2 and 3 we show the relaxation/coupling coefficients for a hadronic gas of pions. The equation of state is taken to be that of a resonance gas of non-interacting pions of mass, $m_\pi = 140 \text{ MeV}$. Knowledge of the above coefficients allows one to write the primary coefficients in terms of the relaxation times. Such relaxation times depend on the collision term in the Boltzmann transport equation, and their derivation is an extremely laborious task.

We now compute the transport coefficient of shear viscosity of a meson gas composed of π, η, ω, ρ and ϕ . We use microscopic transport model, the Ultra-relativistic Quantum Molecular dynamics (UrQMD) [16], using the Green-Kubo formulas. For detailed analysis see [14].

Figure 4 shows the shear viscosity coefficient results from UrQMD using Kubo relations. As in the variational approach the coefficient grows with temperature. The UrQMD results are about twice those from the variational method. This might be due to the many meson resonances included in UrQMD while in the variational method we only have pions. Also the cross section parameterizations are different in the two approaches. Figure 5 shows the relaxation time for shear flux in a hot pion gas calculated from UrQMD by fitting the shear stress correlations. The dependence of the shear relaxation time on temperature is similar to the one obtained using variational method. The results obtained here are about a factor of two less. The reasons are similar to the ones given above for the shear viscosity coefficient. The use of fluctuations through Kubo relations has the advantage of finding not only the transport coefficients but also the corresponding relaxation times. In addition it is also possible to obtain the relaxation coefficients such as β_2 .

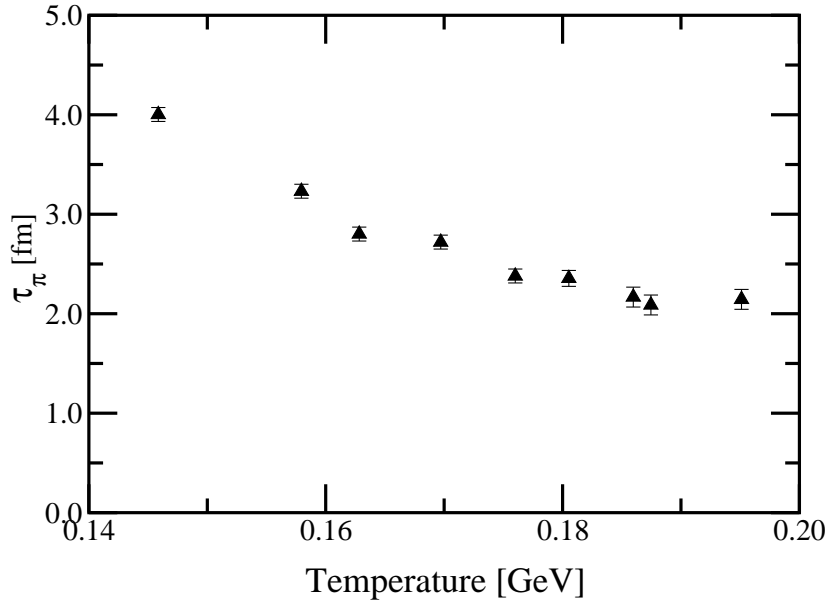


Fig. 5. The relaxation time for the shear flux of meson gas as a function of temperature (cf. [14]).

5 Conclusion

Using the generalized entropy 4-current and the fluctuation theory together with the Green-Kubo formulas we presented the expressions for the standard transport coefficients in terms of the correlations of the dissipative fluxes and the associated relaxation times. The correlations of the dissipative fluxes are related to the non-classical coefficients of the entropy 4-current. An interesting point provided by this analysis is the reduction in the number of independent parameters. In the simple one component relativistic causal fluid dynamics [7] we have eight parameters, namely τ_Π , τ_q , τ_π , ζ , κ , η , $l_{q\Pi}$, $l_{q\pi}$. Equations (12) together with two analogue expressions for the correlations of the heat/viscous couplings provide five relations between these parameters, so that we are left with just three independent parameters, for instance τ_Π , τ_q , τ_π . In this way the macroscopic generalized entropy plus fluctuation theory give much more information than the standard entropy 4-flux with the same number of free parameters.

The Green-Kubo relations are helpful in studying the evolution of the dissipative fluxes. By combining these relations with the generalized entropy expression provides an interesting point of contact between the macroscopic and the microscopic approaches. This is also evident by our choice of using a microscopic transport model, namely UrQMD in extracting the underlying transport coefficients.

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